

The Use of Fourier Descriptor for Geographical Profile Polygon Representations

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Abstract. There is a theoretical and experimental evidence that Fourier analysis is a powerful tool for tackling image processing problems. Its usefulness has been proven through its successful application in several fields. This paper describes the application of a new Fourier descriptor for outlining geographical profiles to be used in reconstruction and recognition.

Keywords: Computer Vision; Image Processing; Fourier Descriptor; Fourier Polygons; Geographical Profiles; Shape Representation; Profile Representation; Outline Profiles.

1. Introduction

Computer vision is the process of deriving useful information from digital images of a scene. The applications of computer vision can be classified in terms of three major areas; compression, enhancement and recognition. Recognition involves the identification of objects in the image which has been generally accepted as one of the most important field in computer vision especially for profile recognition.

This is due to the numerous fields in which profile recognition can be found, such as geographical maps which have been studied extensively over the last two decades and there have been several different approaches to geographical profile recognition problems. The attempts have been made to place geographical profile characterization on a quantitative level by the use of various mathematical, statistical and linguistic descriptions.

A profile can be defined as a list of points defined by a specified starting point and a connected sequence of points which involves stepping point to point around

the boundary leading back to the starting point. The words profile, object, shape, contour and silhouette are sometimes used in the literature interchangeably.

Geographical profile recognition represents a particular problem therefore, the descriptor should be able to handle any type of geographical profile which might contain any type of shape such as sharp corners, straight edges or curves, both concave and convex and independent of translation, rotation, scaling and the starting point. Also that any amount of change made to the geographical profile should correspond to the amount of change in the descriptor features. Fourier descriptors can provide these requirements^[1-4]. Its features provide basic properties of profiles and with flexibility for increasing the number of harmonics in order to get more detail about the profile. The properties of Fourier descriptors ensure that profile representations generated by this technique withstand rotation, translation, scaling, reflection and change of traversal starting point or direction.

2. Fourier Descriptor

The use of Fourier series techniques is now regarded as an importance in the study of many branches of science and engineering^[5-8]. The idea of Fourier theory that all periodic functions can be expressed as a sum of sinusoidal components which permits the interpretation of the series representation of such function as an approximation to the function^[9].

The essential characteristic of a periodic function is the fact that function values repeat regularly at a constant interval of the independent variable *i.e.* the period of the function, $f(x) = F(x+n)$ where n is a positive integer, for all values of x .

The Fourier series for a function of a single variable can be expanded to 2D, *i.e.*, $f(x,y)$. For example, a double Fourier sine series^[10-11],

$$F(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{mpx}{L_1} \sin \frac{np y}{L_2}$$

In order to use this approach for profile recognition, the profile needs to be represented as an image with only two gray level values (black and white), where all points inside the profile are set to black and all other points outside are set to white. The computational effort involved here is quite expensive. Moreover, the normalization is quite complex and difficult to achieve. It is therefore impractical to describe a profile as an image using a 2D Fourier descriptor.

Therefore, in the literature describing 2D profile as 1D in term of Radial and Theta representations has solved the problem. Radial representation defines the profile by measuring the radial length from the center of the profile to each

point on the profile boundary. The Theta representation defines the profile by taking the angular change of each point on the profile boundary against the perimeter length.

However, Radial Fourier descriptor has difficulties in outlining the concave parts of the profile. Therefore, the Radial representation is not sufficient to describe irregular profiles accurately. This problem arises because the Radial representation should, ideally, be uniform along the θ -axis so that the two axis (R, θ) can be moved conveniently relative to one another.

On the other hand, Theta representation in general shows good results for describing regular and irregular profiles, but because the angular representation of a profile is not a smooth function; the number of harmonics used for Fourier Descriptor has to be very high in order to outline the profile accurately.

A new technique is developed using Fourier series based on computing the two parameters (x, y) separately^[12], this makes it possible to use a $1D$ Fourier series algorithm to perform a $2D$ profile. The idea that in image processing, each point has a unique set of values (x, y) where the value of x or y of a given point is related to any of its neighbors by the addition or subtraction of 1 or 0 . As a consequence, the functions which describe the profile in terms of x and y separately, are very smooth in comparison with other representation of profile such as Theta and Radial profile representations.

This makes it possible to use a $1D$ Fourier series to perform a $2D$ profile. However, Fourier series can only represent a function of period 2π since sine and cosine functions each has a period of 2π . Therefore, in order to use Fourier analysis, a profile has to be represented as a periodic function. In image processing, a profile is represented by a series of points from 1 to N , where N is the total number of points in the profile.

Therefore, in Fourier terms, each profile can be said to have a period of N since one complete passage around the profile returns to the start point, so we can convert N into an interval of 2π . This approach gives a considerable saving in computation time and can outline profiles sufficiently using only a few harmonics. Therefore, the computational cost for profile representation is considerably lower compared to traditional Fourier transforms descriptors.

The basis of $1D$ Fourier series is to represent a function $f(u)$ by a trigonometric series of the form^[10,11],

$$F(x) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} \{A_n \cos nx + \sin nx\}$$

Where the Fourier coefficients numerical values (A_0 , A_n and B_n) are defined by equations based on the original function. The coefficients A_0 , A_n and B_n represent the magnitude of the harmonic components and provide information about the frequency content.

3. Fourier Descriptor Features

The XY Fourier descriptor has the potential for geographical profile recognition in many different ways. Its coefficient features provide basic properties of profiles with flexibility for increasing the number of harmonics in order to get more features which could describe the profile in more detail. Its outlining of profiles provides powerful features for modeling and matching of profiles where profile matching is powerful techniques for geographical profile recognition in particular it is effective for inspecting geographical profiles with defects.

The matching can be carried out as global or local. Global matching quantifies the similarity between the entire geographical profile and the entire geographical model, whilst local matching quantifies the similarity between portions of the geographical profile and of the model.

Another advantage of using the XY Fourier descriptor is that a profile can be reconstructed as a polygon representation which offers a compact representation. In practice, the goal of a polygonal approximation is to capture the "essence" of the geographical profile with the fewest possible polygon segments. This allows the XY Fourier descriptor to be used for finding the critical points of the geographical profile.

Figures 1(a,b) and 2(a,b) show geographical profiles where their size (N) are represented with 2, 3, 4, 5, 10, 50, 100, 250, and 500 points joined by line segments using 8 harmonics. Observing the figures, one can see that each geographical profile is reduced to particular points and still contain the most significant features of the original geographical profile.

The polygon representation appears to offer good prospects for classifying profiles and can be described as a compact features which offer distinctive patterns for geographical profiles where it can be used efficiently to identify a particular type of geographical profile. The essential details of the profile are described by the low-order polygon representation whereas higher polygon representation describes the profile in greater detail. This polygon representation technique of using XY Fourier Descriptor can be very useful in application where profiles are very large and only the most significant features are needed to be stored and documented which means the reduction of unnecessary information on the profile.

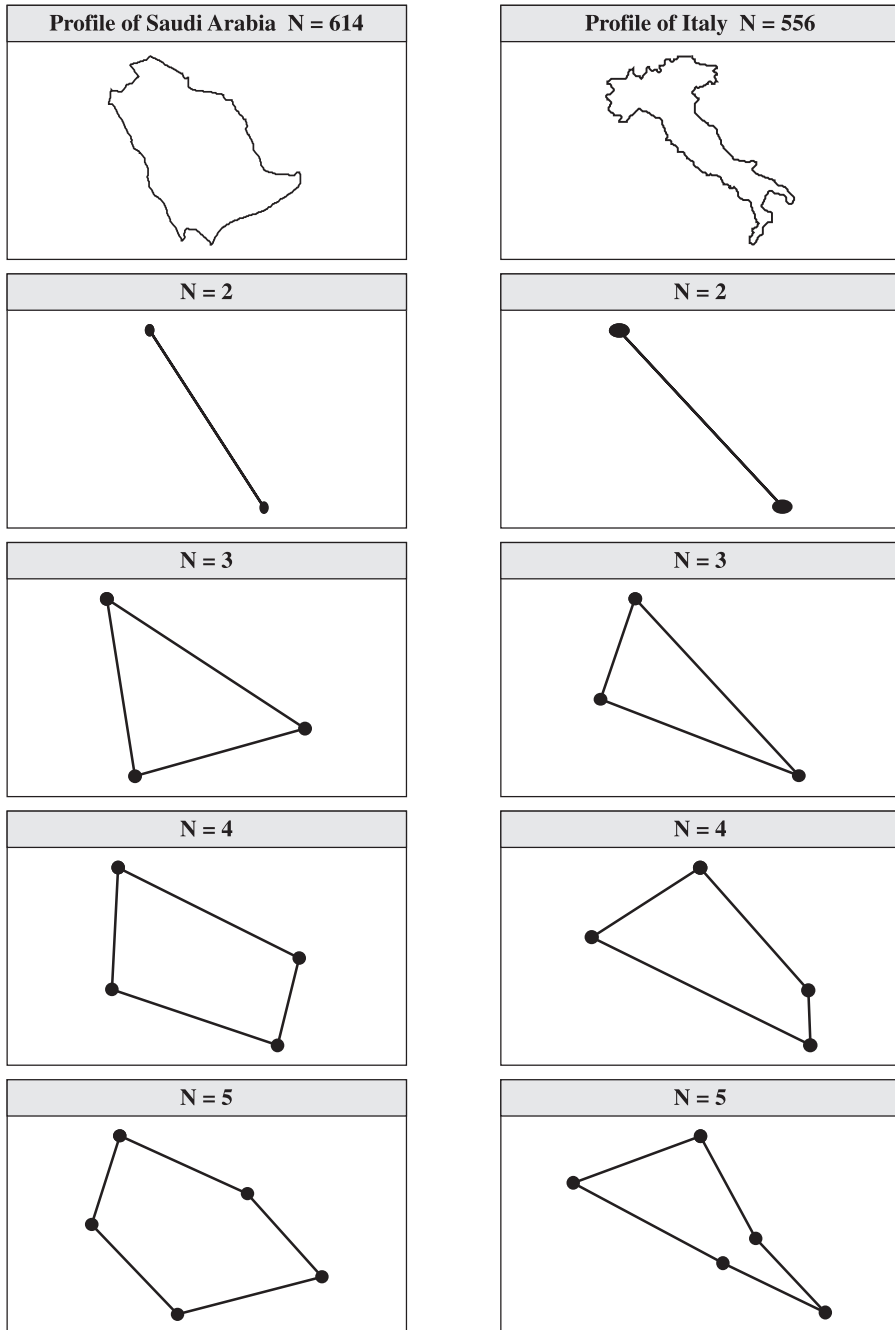


Fig. 1(a). XY Fourier descriptor with different sizes (N = 2,3,4 and 5).

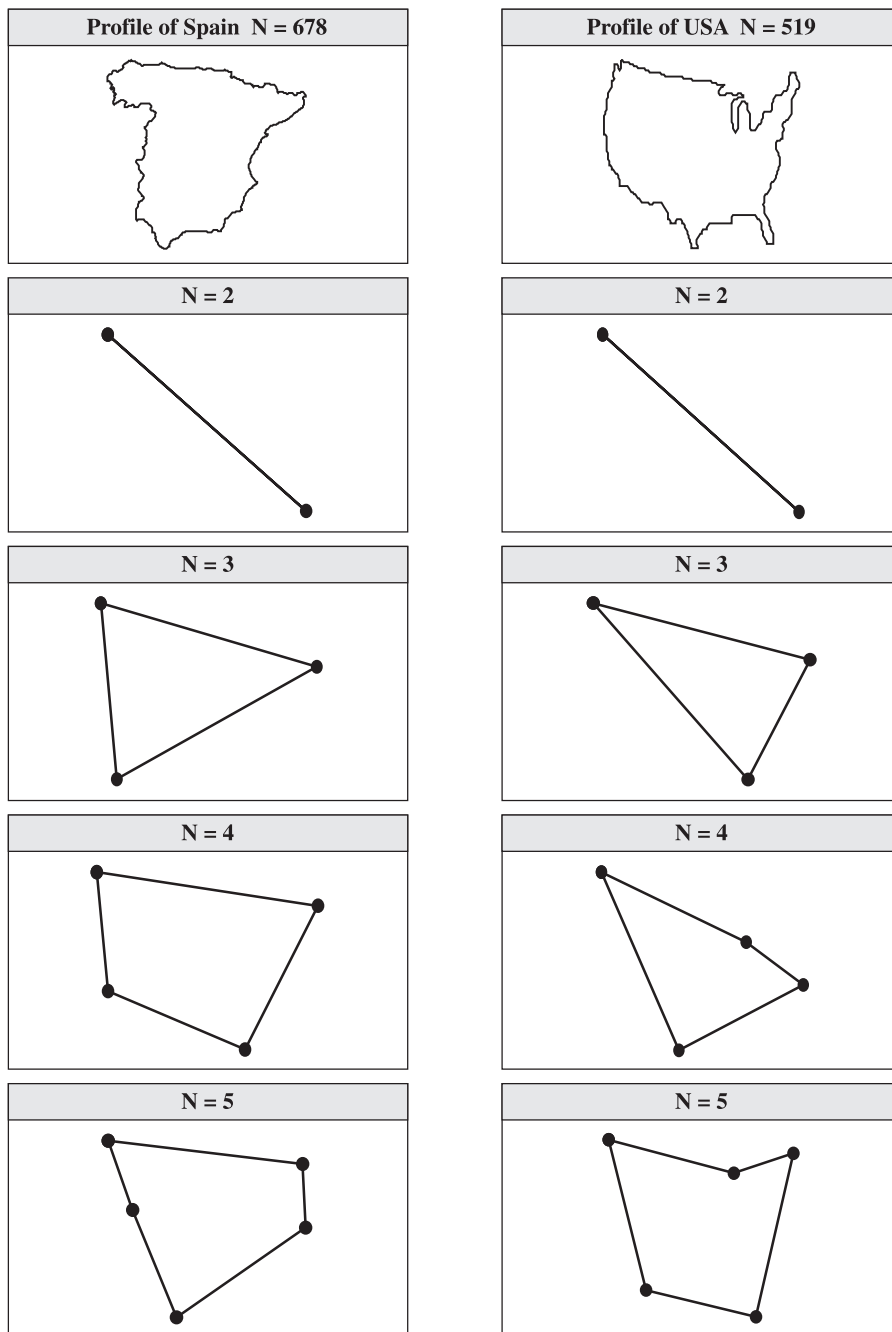


Fig. 1(b). XY Fourier descriptor with different sizes ($N = 2,3,4$ and 5).

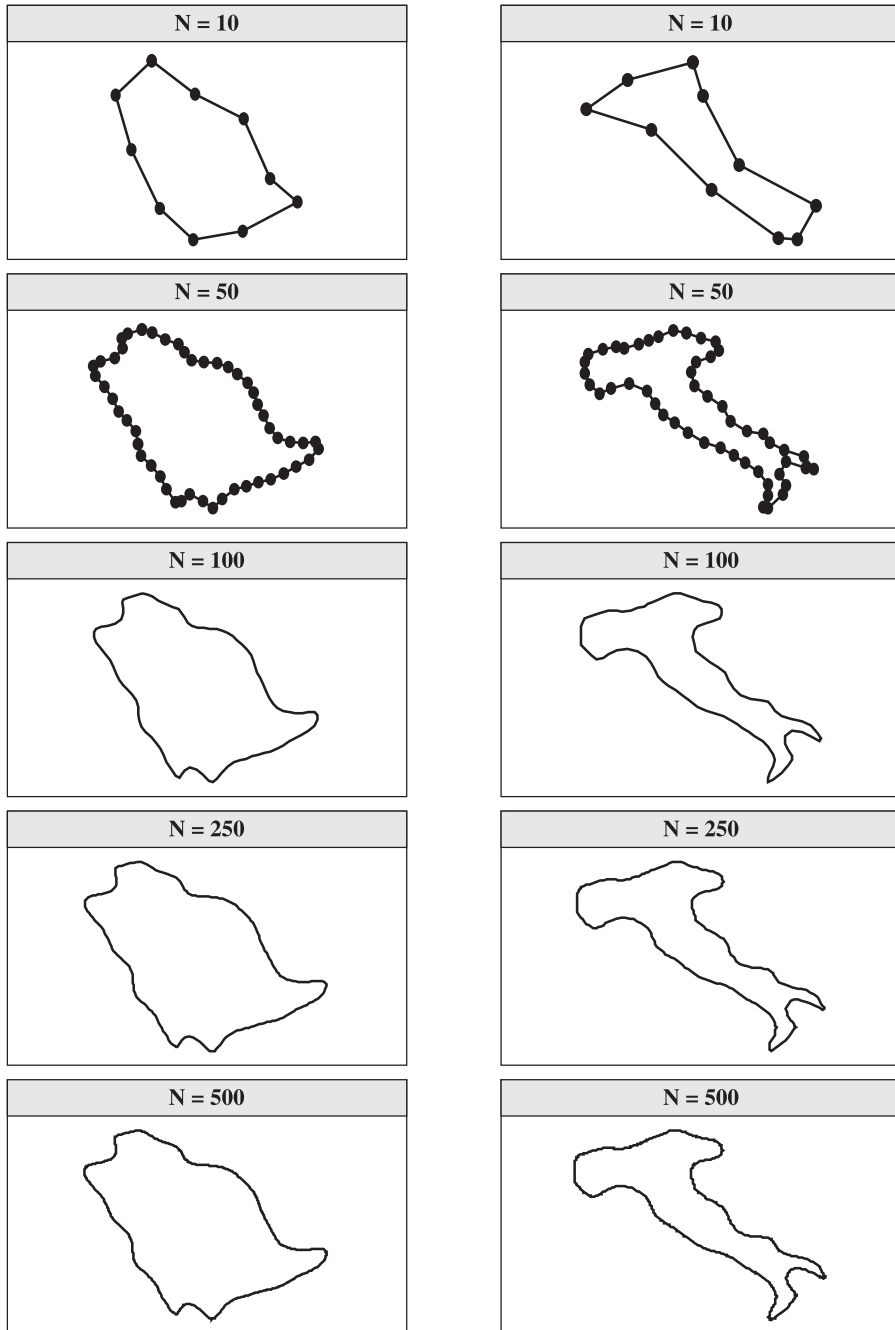


Fig. 2(a). XY Fourier descriptor with different sizes (N = 10,50,100,250 and 500).

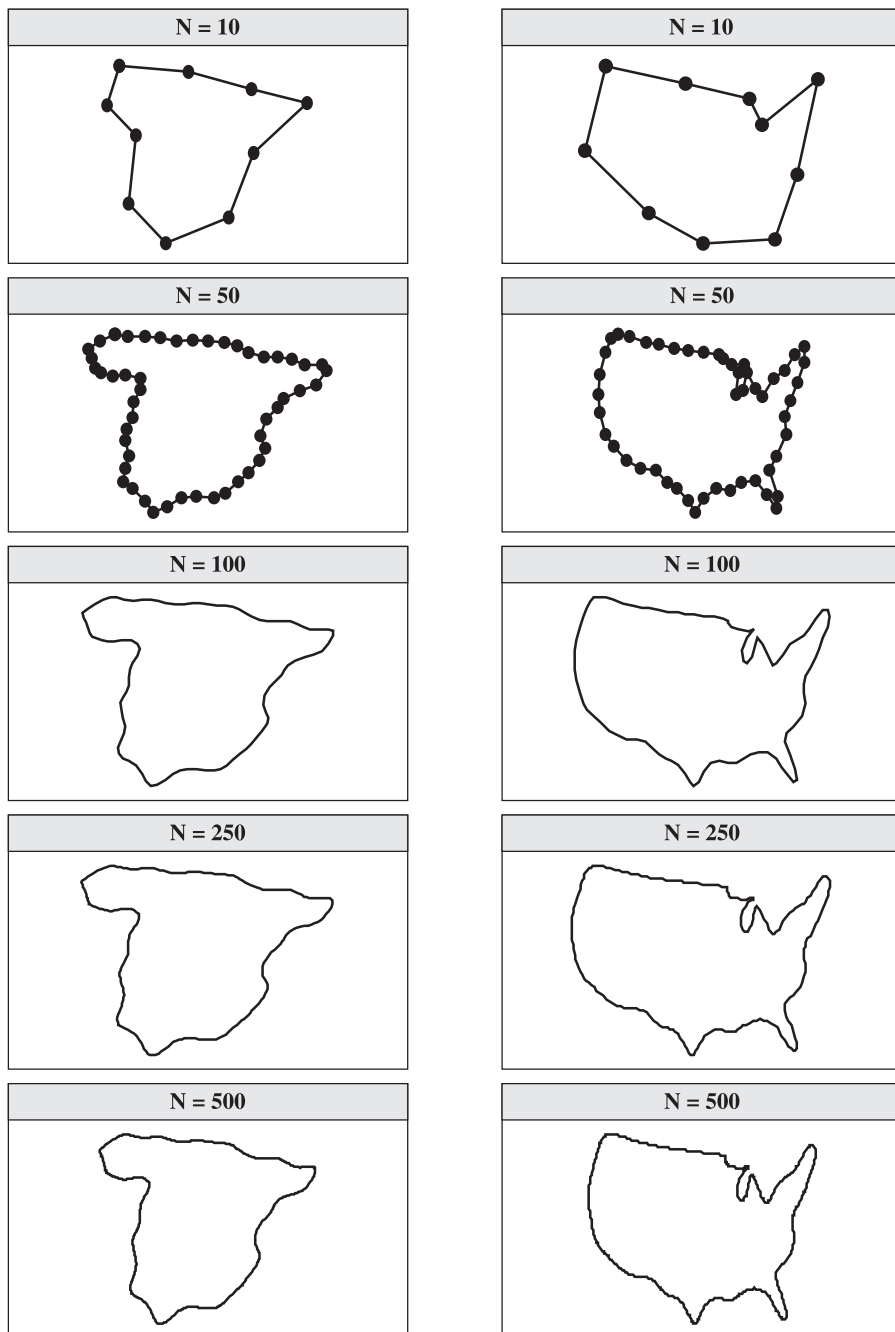


Fig. 2(b). XY Fourier descriptor with different sizes ($N = 10, 50, 100, 250$ and 500).

4. Conclusion

This paper reports on the development of XY Fourier Descriptor which can outline profiles very accurately and give considerable saving in computation time. The polygon representation using Fourier Descriptor provides powerful features for modeling and matching of geographical profiles and can be successfully detected even after rotation, translation, scaling and reflection of the host geographical profile. Future work will concentrate on the enhancement and restoration of remotely sensed satellite images of outer space using XY Fourier Descriptor which will lead to the automation of profile classification and identification.

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استخدامات الواصف فوريا لتحليل الخطوط الخارجية للخرائط الجغرافية

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المستخلص. يوجد أدلة نظرية وتطبيقية على قوة Fourier Descriptor كوسيلة في معالجة العديد من المشاكل الموجودة في التحليل التصويري عن طريق الحاسب الآلي. وكثير من الفوائد المستفادة من Fourier Descriptor تم البرهنة عليها من خلال الاستخدامات التي تمت في عدة مجالات علمية مختلفة. يتطرق هذا المقال إلى تطبيق نظام جديد يعتمد على تقنية Fourier Descriptor في تحليل الخرائط الجغرافية، وإمكانية تمثيل الخريطة الجغرافية في عدد محدود من النقاط، والتي يمكن من خلالها التعرف على الخريطة، بالإضافة إلى عدة فوائد أخرى منها التخزين والتحسين. وهذه التقنية لا تتأثر بجميع الحالات التي يمكن أن تحدث للخريطة، مثل تغير الحجم أو الموقع للصورة.